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TECHNICAL MEMORANDUM

No. 1151

LIFT AND DRAG OF WINGS WITH SMALL SPAN*

By F. Weinig

Translation

“Auftrieb und Widerstand des Tragflügels kleiner Spannweite”

Deutsche Luftfahrtforschung, Forschungsbericht Nr. 1665



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LIFT AND DRAG OF WINGS WITH SMALL SPAN*

By F. Weinig

Abstract: The lift coefficient of a wing of small span at first shows a linear increase for the increasing angle of attack, but to a lesser degree than was to be expected according to the theory of the lifting line; thereafter the lift coefficient increases more rapidly than linearly, as contrasted with the theory of the lifting line. The induced drag coefficient for a given lift coefficient, on the other hand, is obviously much smaller than it would be according to this theory. A small change in the theory of the lifting line will cover these deviations.

Outline:

1. Symbols
2. The Previous Treatment of the Wing of Small Span
3. New Treatment Based upon an Enlarged Array of Free Vortices
4. Summary
5. Bibliography

1. SYMBOLS

A	lift
F	wing area ($F = \frac{\pi}{4} b \bar{t} = \text{Wing area for elliptic base}$)
t	wing chord at any point
\bar{t}	wing chord at center of the wing
b	span
a	gap of the cascade which is equivalent to the wing profile with respect to the deflected mass (compare equation (8))
v	flight velocity

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Nr. 1665, Stuttgart, Sept. 8, 1942.

$\frac{\rho}{2} v^2$	stagnation pressure
$v_{1\infty}$	induced downwash velocity far behind the wing
v_1	induced downwash velocity at the location of wing $\left(\frac{v_{1\infty}}{2} \right)$
q	deflection coefficient of cascade equivalent to the wing profile (compare equation (13) $\left(\tan \frac{\pi t}{2a} \right)$
\dot{m}	mass deflected by wing per unit time
ρ	air density
α	angle of attack
α_1	induced angle of attack $\left(\arctan \frac{v_1}{v_\infty} \right)$
Λ	aspect ratio, b^2/F $\left(\Lambda = \frac{4}{\pi} \frac{b}{t} = \text{Aspect ratio of elliptic wing} \right)$
c_a	lift coefficient according to modified theory, (c_a) lift coefficient according to theory of lifting line
c_{w1}	induced drag coefficient according to modified theory, (c_{w1}) induced drag coefficient according to theory of lifting line
Γ	circulation at any point
$\bar{\Gamma}$	circulation at the center of wing
ζ_a	ratio of lift according to modified theory to lift according to theory of lifting line $(c_a/(c_a))$
ζ_w	ratio of induced drag coefficient according to modified theory to value according to theory of lifting line $(c_{w1}/(c_{w1}))$

2. THE PREVIOUS TREATMENT OF THE WING OF SMALL SPAN

The comparatively simple theory of the lifting line leads to satisfactory results for the wing with an aspect ratio b/t of

sufficient magnitude at least in the case of unseparated flow, but for the wing with small aspect ratio the results with respect to lift and induced drag are quite unsatisfactory (1), (2). A simple modification of this theory (3), (4) brought useful results at least for very small angles of attack. The transition to the theory of the lifting surface (5), (6) or to the theory of the acceleration potential which methodically replaces the latter theory (7), (8), (9), (10) also provided such results for the range of very small angles of attack. But these theories disappointed for greater angles of attack, even while the flow was not yet separated.

It is true the simple modification of the theory of the lifting line which was mentioned above gave a connection between lift and induced drag coefficient which seemed still useful for the domain of larger lift coefficient even though the relation between lift and angle of attack itself was not clarified (11).

For small angles of attack the lift at first increases linearly with the angle of attack, but the increment grows with increasing angles of attack when the aspect ratio is small, and the smaller the aspect ratio the more noticeable is this influence (12). Likewise the induced drag at small aspect ratios is obviously considerably smaller than resulted from the original theory of the lifting line (13).

The conjecture was brought forward that an explanation for these conditions might perhaps be found in the fact that the vortex area leaving the wing was somewhat wider than the span (14). Hereby one could explain the reduction of the induced drag, but not the behavior of the lift coefficient. Neither can the downwash angle sufficiently far behind the wing, on the other hand, be larger than the angle of attack, at the most.

3. NEW TREATMENT BASED UPON AN ENLARGED ARRAY OF FREE VORTICES

There is to be found, however, a natural explanation for the actual behavior of the wing of small span: One has to start from the fact that the effective mass which is deflected downward by the wing is greater than resulted from the original theory of the lifting line for elliptic lift distribution. according to this theory, this mass would correspond to the mass of a cylinder of air density with a diameter equivalent to the span. However, the path of a wing must no longer be treated as a flat plane as it was in the case of a large span but rather as a prism with the cross

section $F \sin \alpha$. The mass contained therein, however, must also be deflected downward¹, at least approximately. (See fig. 1.)

With $v_{i\infty}$ as downwash velocity far behind the wing and with

$$\dot{m} = \rho v \left(\frac{\pi}{4} b^2 + F \sin \alpha \right) \quad (1)$$

as the mass deflected downward per unit time the lift will be, according to the momentum theorem,

$$A = \dot{m} v_{i\infty} = \rho v \left(\frac{\pi}{4} b^2 + F \sin \alpha \right) v_{i\infty} \quad (2)$$

Or, with

$$v_i = \frac{v_{i\infty}}{2} = v \tan \alpha_i \quad (3)^2$$

¹Mangler (15) shows another way of treatment of this influence. According to his theory, the effect upon the outside flow of the lateral rolling up of the vortex sheet can be replaced by end plates, but this way leads to essentially equivalent results to the one which was used here. Furthermore we should like to point out the treatment by W. Bollay (16); there also the free vortices behind the wing do not lie in a plane. However, the results of the present treatment are much simpler and at least as satisfactory; also they are more readily applicable to any shape of wings.

²It is a matter of indifference whether

$$v_i = \frac{v_{i\infty}}{2} = v \tan \alpha_i \text{ or } v_i = \frac{v_{i\infty}}{2} = v \sin \alpha_i$$

and

$$\tan \alpha_i = \frac{q}{1+q} \tan \alpha \text{ or } \sin \alpha_i = \frac{q}{1+q} \sin \alpha$$

is introduced for small angles of attack. For larger angles of attack, however, better agreement of the results of the calculation and the measurement is obtained by the use of the first-mentioned form only.

as effective downwash velocity at the location of the wing and the aspect ratio

$$\frac{b^2}{F} = \Lambda \quad (4)$$

$$A = \frac{\rho}{2} v^2 F \left(\frac{\pi}{4} \Lambda + \sin \alpha \right) 4 \tan \alpha_1 \quad (5)$$

A lift coefficient of

$$c_a = \tan \alpha_1 \left(\pi \Lambda + 4 \sin \alpha \right) = 2\pi \tan \alpha_1 \left(\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha \right) \quad (6)$$

would result. In this equation,

$$\tan \alpha_1 = \frac{q}{1 + q} \tan \alpha \quad (7)^3$$

The deflection coefficient q results from the following consideration (compare (3) and (4)): To each wing element a flow domain directly influenced by it is coordinated; this flow domain is bounded in a first approximation by streamlines as figure 2 shows. This flow domain is replaced by a strip whose width equals the width of the wing element and whose length is such as to make the kinetic energy conveyed by the wing element equivalent to the quantity of energy which is conveyed to the coordinated flow domain. The length of this strip will be

$$a = \frac{\Gamma}{v_{1\infty}} + t \sin \alpha \quad (8)$$

Accordingly, the wing profile influences the coordinated flow like an identical profile arranged in a cascade (fig. 3) of a stagger of 90° and a gap a . With

$$\begin{aligned} \Gamma &= \bar{\Gamma} \sin \vartheta \\ t &= \bar{t} \sin \vartheta \end{aligned} \quad \left(\cos \vartheta = \frac{y}{b/2} \right) \quad (9)$$

³Same as footnote 2.

the gap ratio becomes

$$\frac{a}{t} = \frac{\Gamma}{t v_{1\infty}} + \sin \alpha = \frac{\bar{\Gamma}}{t v_{1\infty}} + \sin \alpha \quad (10)$$

or, for elliptic lift and circulation distribution, respectively, with

$$v_{1\infty} = \frac{\Gamma}{t v_{1\infty}} \quad \Lambda = \frac{4}{\pi} \frac{b}{t} \quad (11)$$

$$\frac{a}{t} = \frac{b}{t} + \sin \alpha = \frac{\pi}{4} \Lambda + \sin \alpha \quad (12)$$

The value q becomes, according to the results of the theory of the cascade flow,

$$q = \tan \frac{\pi}{2} \frac{t}{a} = \tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha} \quad \checkmark \quad (13)$$

and therefore

$$\tan \alpha_1 = \frac{\tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}}{1 + \tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}} \tan \alpha \quad (14)$$

As a lift coefficient, there results

$$c_a = 2\pi \frac{\tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}}{1 + \tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}} \left(\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha \right) \tan \alpha \quad (15)$$

The induced drag coefficient becomes

$$c_{w_1} = c_a \tan \alpha_1 = 2\pi \left(\frac{\tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}}{1 + \tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}} \right)^2 \left(\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha \right) \tan^2 \alpha \quad (16)$$

or

$$c_{w_1} = \frac{c_a^2}{\pi \Lambda + 4 \sin \alpha} \quad (16a)$$

There results the ideal normal-force coefficient

$$c_n = c_a \cos \alpha + c_{w_1} \sin \alpha$$

$$c_n = 2\pi \frac{\tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}}{1 + \tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}} \left(\frac{\Lambda}{2} + \frac{2}{\pi} \right)$$

$$\sin \alpha \left(1 + \frac{\tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}}{1 + \tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}} \frac{\sin^2 \alpha}{\cos \alpha} \right) \quad (17)$$

In the boundary case of very small span there results for small angles of attack

$$c_n / \Lambda \rightarrow 0 = 2 \sin^2 \alpha \quad (17a)$$

In the case of large span, the result for small angles of attack is,

$$c_n / \Lambda \rightarrow 0 = 2\pi \sin \alpha \cos \alpha \quad (17b)$$

In figures 4(a), 4(b) and 5(a), 5(b) the results of this calculation are represented for comparison with test results (12): figures 4(a) and 4(b) show the polars $c_a(c_w)$ according to test and $c_a(c_{w1})$

according to calculation, respectively, (figs. 5(a) and 5(b)) the normal force coefficients $c_n(\alpha)$ as a function of the angle of attack according to test (fig. 5(a)) and to calculation (fig. 5(b)). The agreement is relatively good. The deviations are probably caused by the influence of friction.

From the original theory of the lifting line there results the lift coefficient

$$(c_a) = \frac{2\pi}{1 + \frac{2}{\Lambda}} \sin \alpha \quad (18)$$

and the induced drag coefficient, for a given c_a ,

$$(c_{w1}) = \frac{c_a^2}{\pi \Lambda} \quad (19)$$

The resulting ratios are, for the lift coefficient,

$$\zeta_a = \frac{c_a}{(c_a)} \frac{\tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}}{1 + \tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}} \frac{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}{\cos \alpha} \left(1 + \frac{2}{\Lambda}\right) \quad (20)$$

and for the induced drag coefficient, for a given lift coefficient,

$$\zeta_w = \frac{c_{w1}}{(c_{w1})} = \frac{\pi \Lambda}{\pi \Lambda + 4 \sin \alpha} \quad (21)$$

Figure 6 shows the value ζ_a for various ratios Λ and angles of attack α .

The value ζ_w as a function of c_a for various ratios is represented in figure 7(b). The relation between the value c_a and the ratio Λ was determined therefrom; figure 8(b) shows this relation for various lift coefficients.

The value ζ_w can also be determined by use of the test results represented in figures 4(a) and 5(a) with the resulting relation between angle of attack α and lift coefficient, and by means of the relations

$$c_{w1} = c_a \tan \alpha_1 = c_a \tan \alpha \frac{\tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}}{1 + \tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}} \approx c_a \tan \alpha \frac{\tan \frac{1}{\Lambda/2}}{1 + \tan \frac{1}{\Lambda/2}}$$

$$c_{w1} = \frac{c_a^2}{\pi \Lambda}$$

There results a value

$$\zeta_w = \frac{\pi\Lambda}{c_a} \frac{\tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}}{1 + \tan \frac{1}{\frac{\Lambda}{2} + \frac{2}{\pi} \sin \alpha}} \tan \alpha \approx \frac{\pi\Lambda}{c_a} \frac{\tan \frac{1}{\Lambda/2}}{1 + \tan \frac{1}{\Lambda/2}} \tan \alpha \quad (21a)$$

The approximation indicated above was used for the evaluation; the values neglected in this approximation were very small. Based upon the connection between c_a and α found by test, the relations represented in figures 7(a) and 8(a) are determined.

The comparison of figures 7(a) and 8(a) with figures 7(b) and 8(b) shows certain differences which also may probably be traced mainly to influences of friction. However, these differences are still so small as to be negligible.

For large aspect ratios also there results an influence upon lift coefficient and induced drag coefficient within the range of the usual angles of attack. The influence on the lift coefficient will, generally, be negligible; however, such a neglect might not always be permissible for the induced drag coefficient; for $\Lambda \gtrsim 5$ c_{wi} may be estimated

$$c_{wi} = \frac{c_a^2}{\pi\Lambda} \left(1 - \frac{2c_a}{\pi^2\Lambda} \right).$$

The results were derived for untwisted wings with elliptic base planes; however, the proportionality factors ζ_a and ζ_w will probably furnish results which can also be applied to wings of different design as shown by a comparison with the test results for rectangular wings.

4. SUMMARY

The lift coefficient and the induced drag coefficient for a wing of small span may be determined with sufficient accuracy, as

long as the flow is not separated, in the following way: A three-dimensional array of free vortices which corresponds to the angle of attack is taken as a base and a finite flow domain is coordinated to each wing element. The theory of the lifting line will have to be changed slightly. Thereby an explanation will, in particular, be given for the rapid, nonlinear increase of the lift coefficient which corresponds to an increase of the angle of attack, and for the relative smallness of the induced drag as compared with the results of the unmodified theory of the lifting line. These results will have special importance for the calculation of the lift of the fuselage and of the lift and drag of vertical tail surfaces.

Translated by Mary L. Mahler
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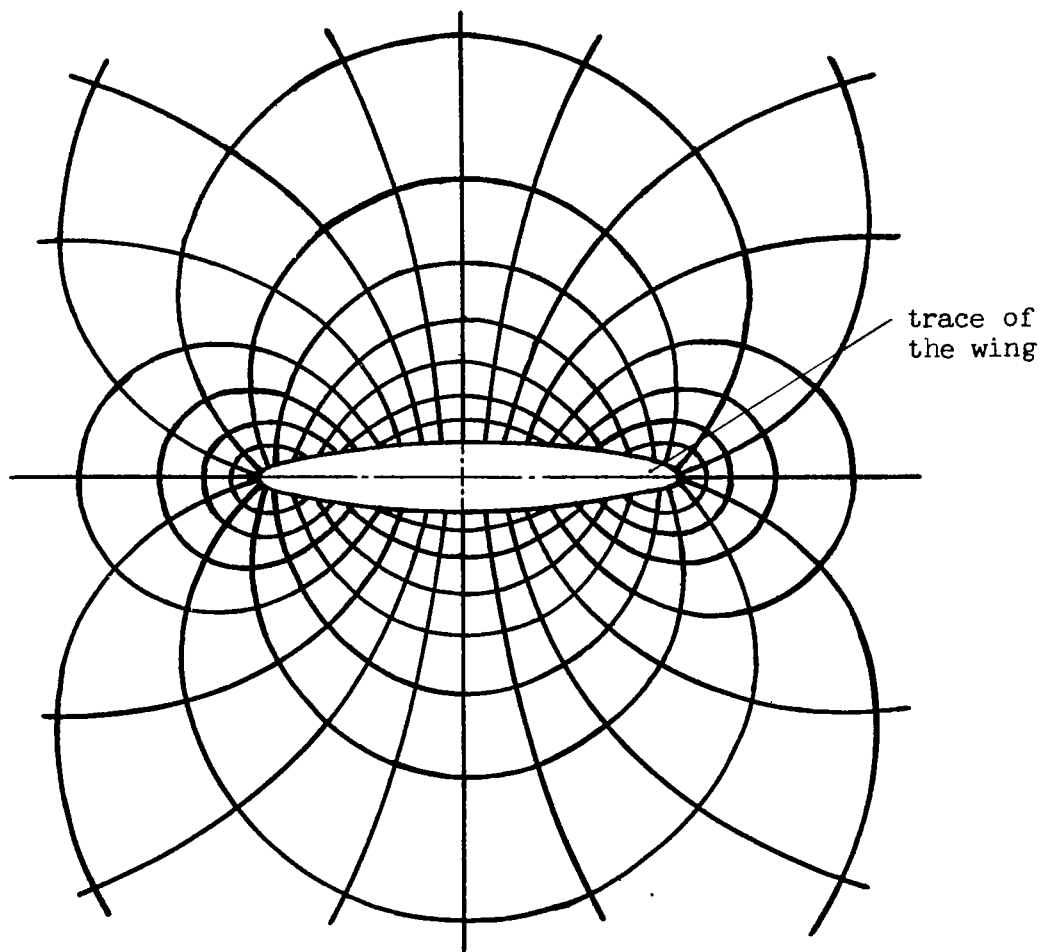


Figure 1. Flow about array of vortices behind an elliptic wing of small span.

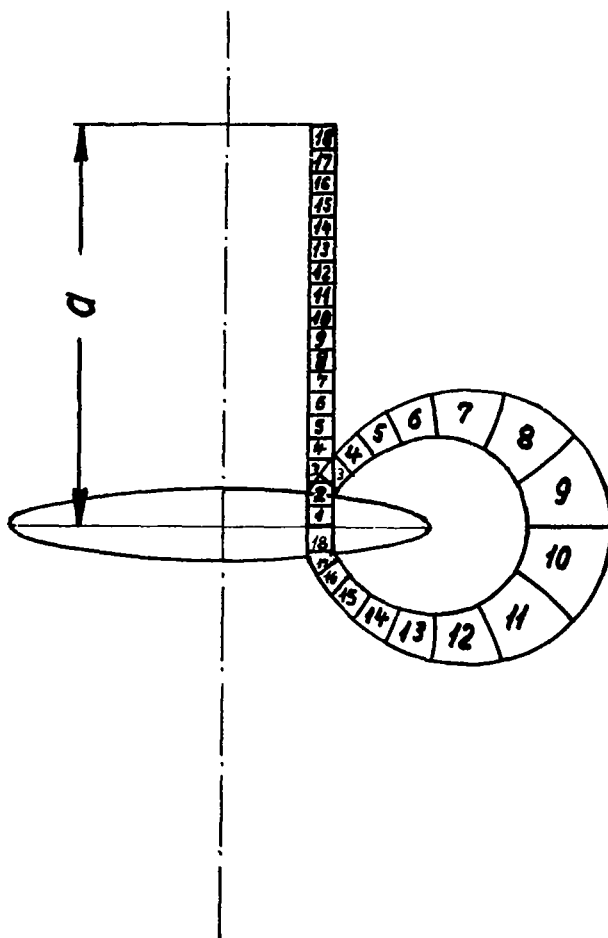


Figure 2. The flow domain influenced by a wing element.

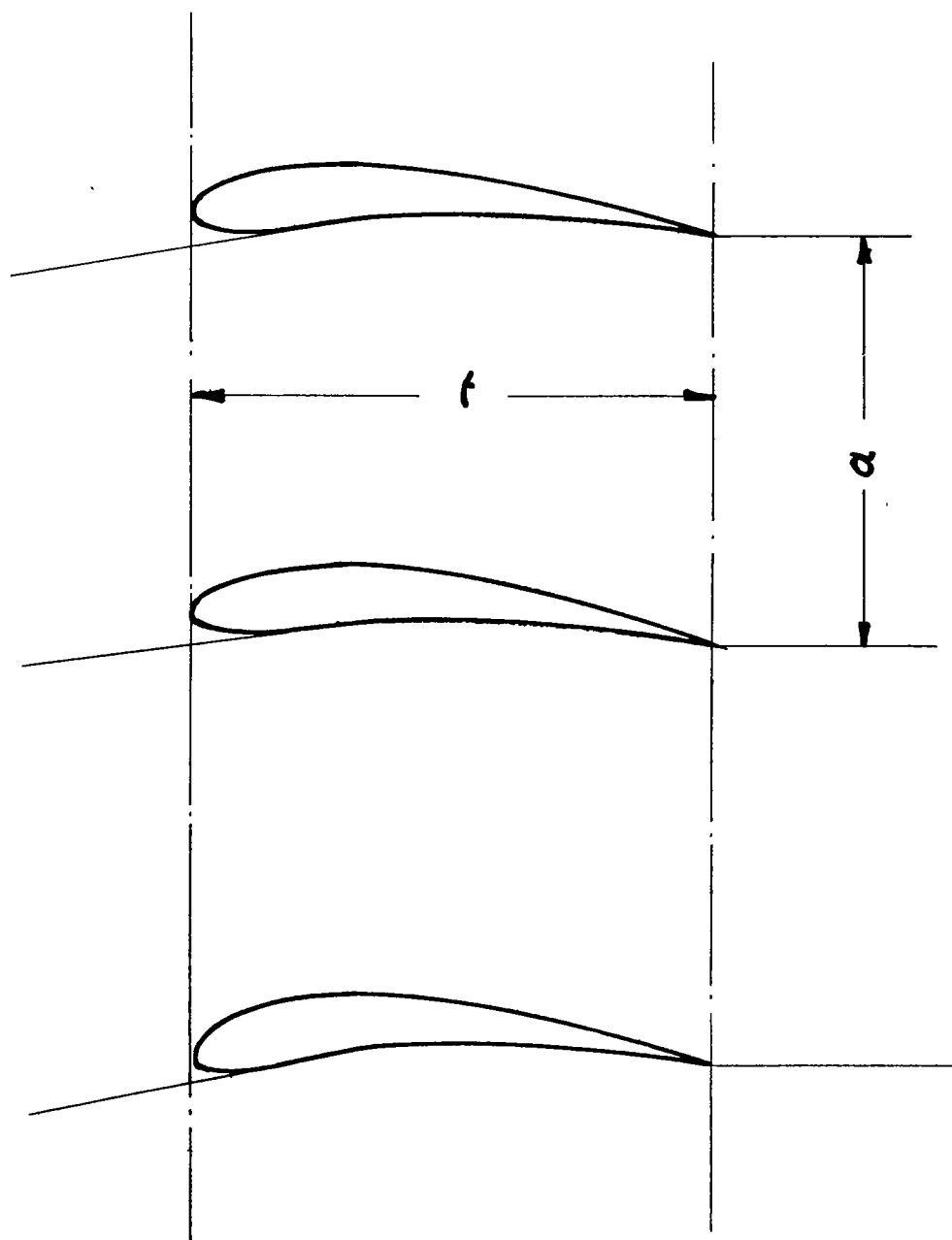


Figure 3. The profile cascade which replaces the wing profile in its effect.

Fig. 4a

NACA TM No. 1151

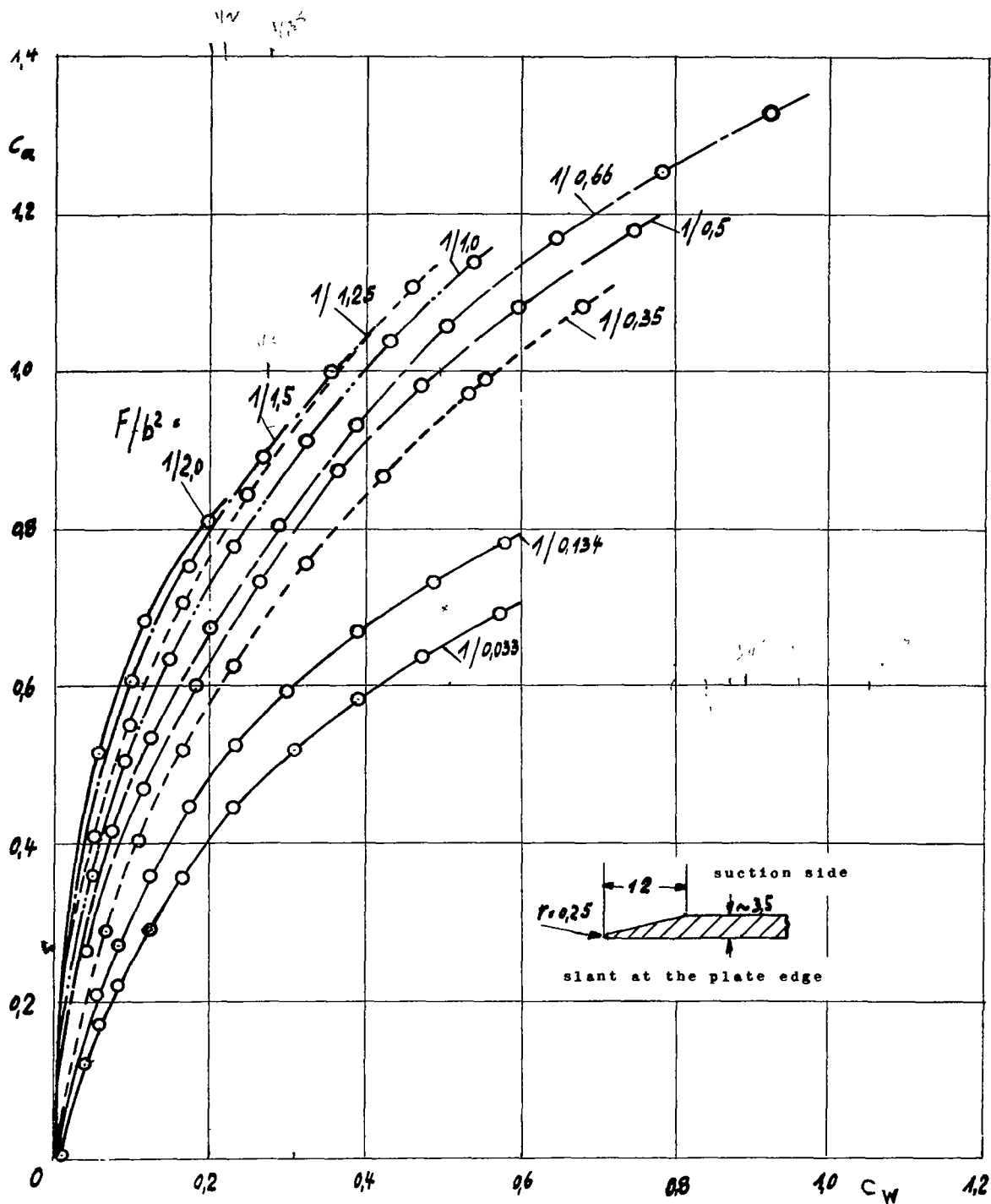


Figure 4a. Polars for wings with small aspect ratio. Drag and lift coefficient according to measurement.

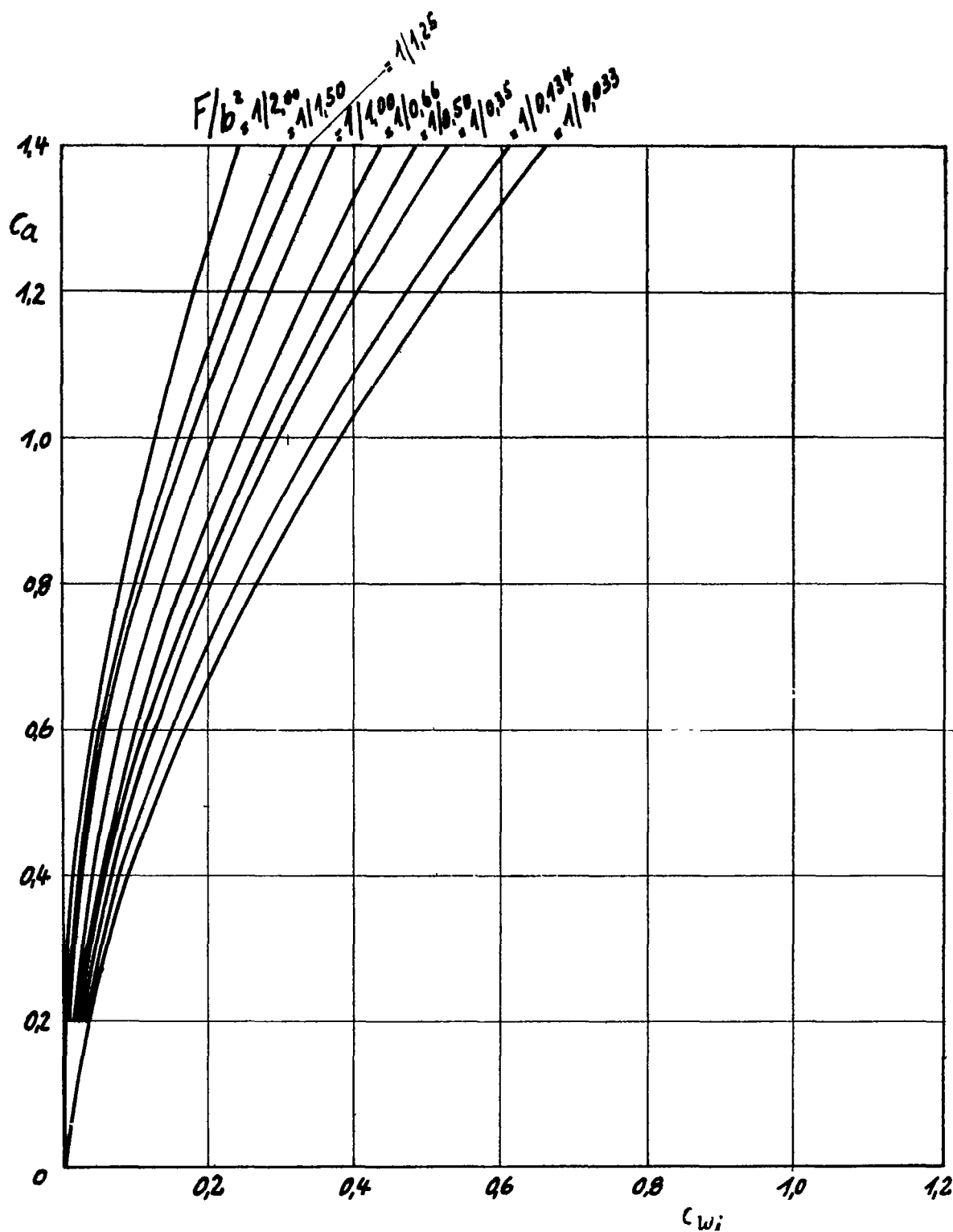


Figure 4b. Polars for wings with small aspect ratio. Induced drag coefficient and lift coefficient according to calculation.

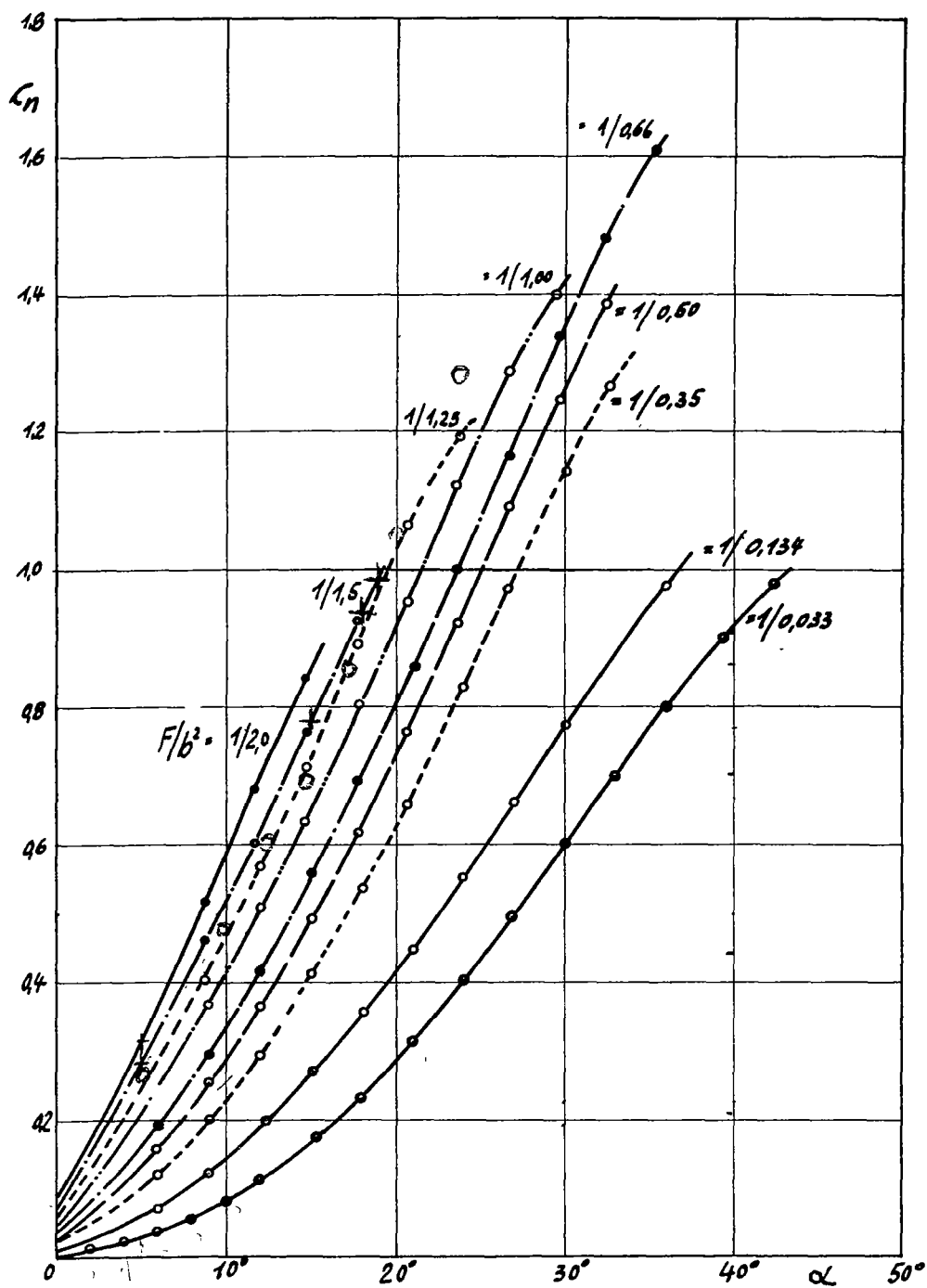


Figure 5a. Normal force coefficient for wing with small aspect ratio as a function of the angle of attack according to measurement.

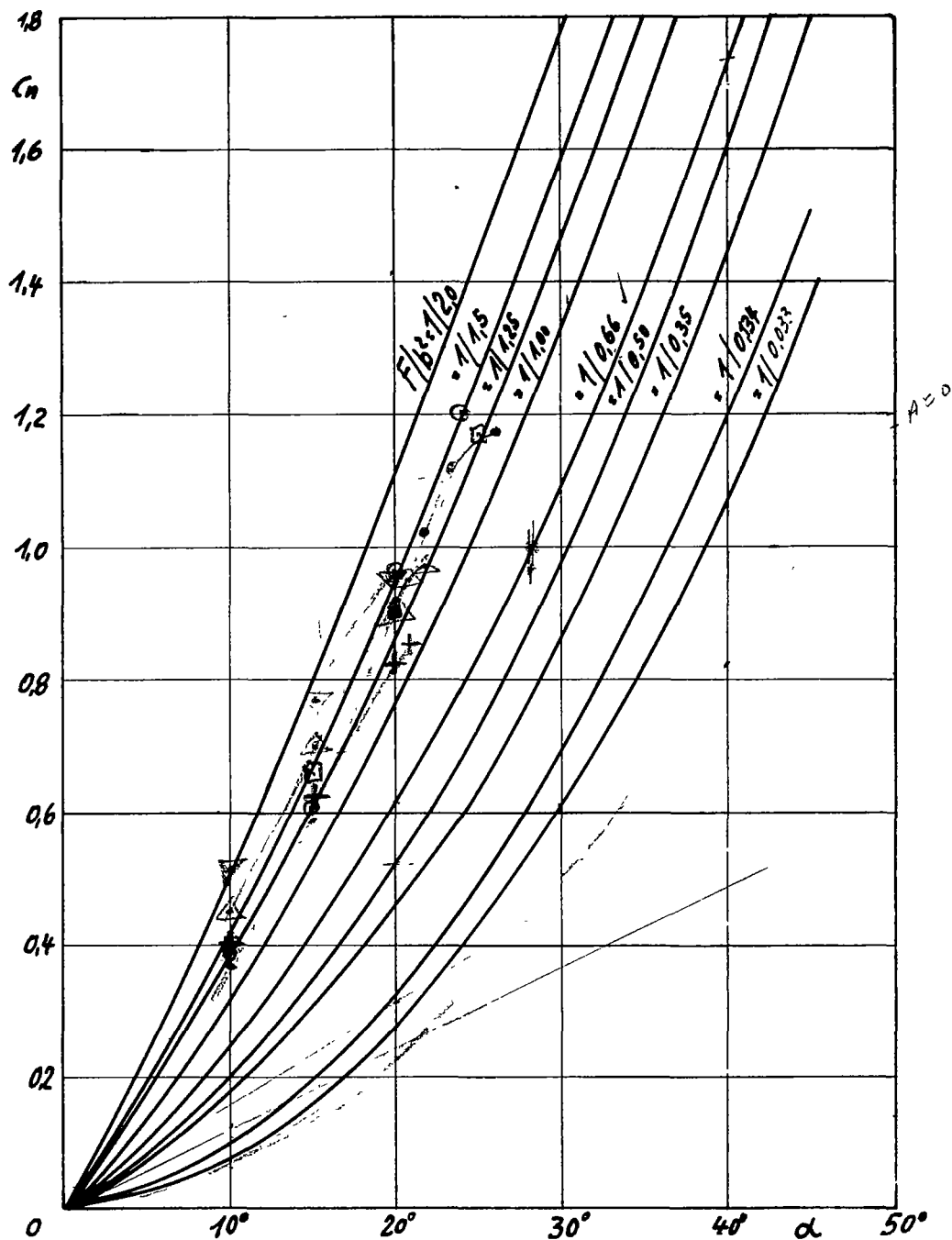


Figure 5b. Normal force coefficient for wing with small aspect ratio as a function of the angle of attack according to calculation.

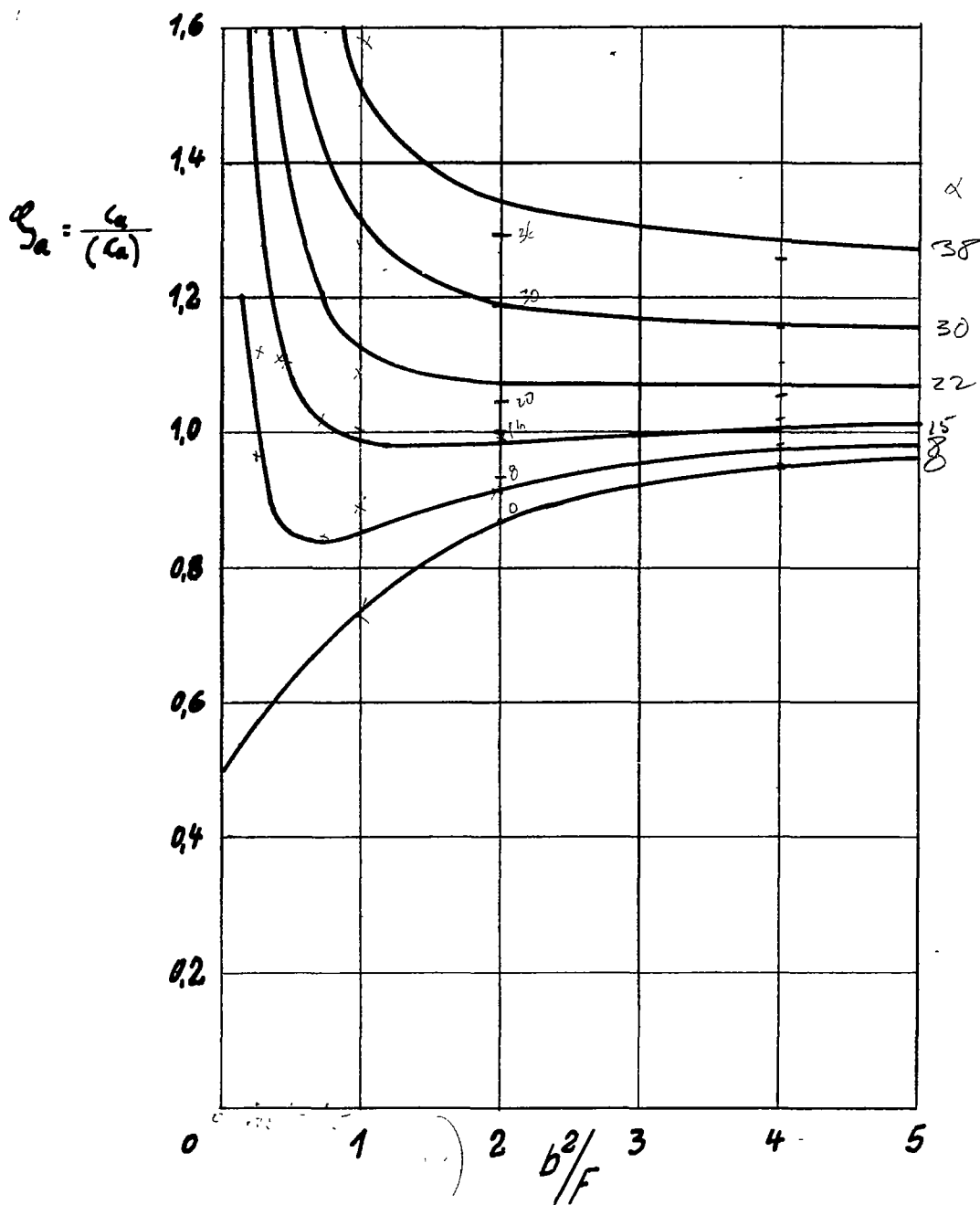


Figure 6.- Ratio $\zeta_a = \frac{c_a}{(c_a)}$ (ratio of the lift coefficient c_a , according to the modified theory for small span, to the lift coefficient (c_a) , according to the theory for large span wings) as a function of the aspect ratio for various angles of attack.

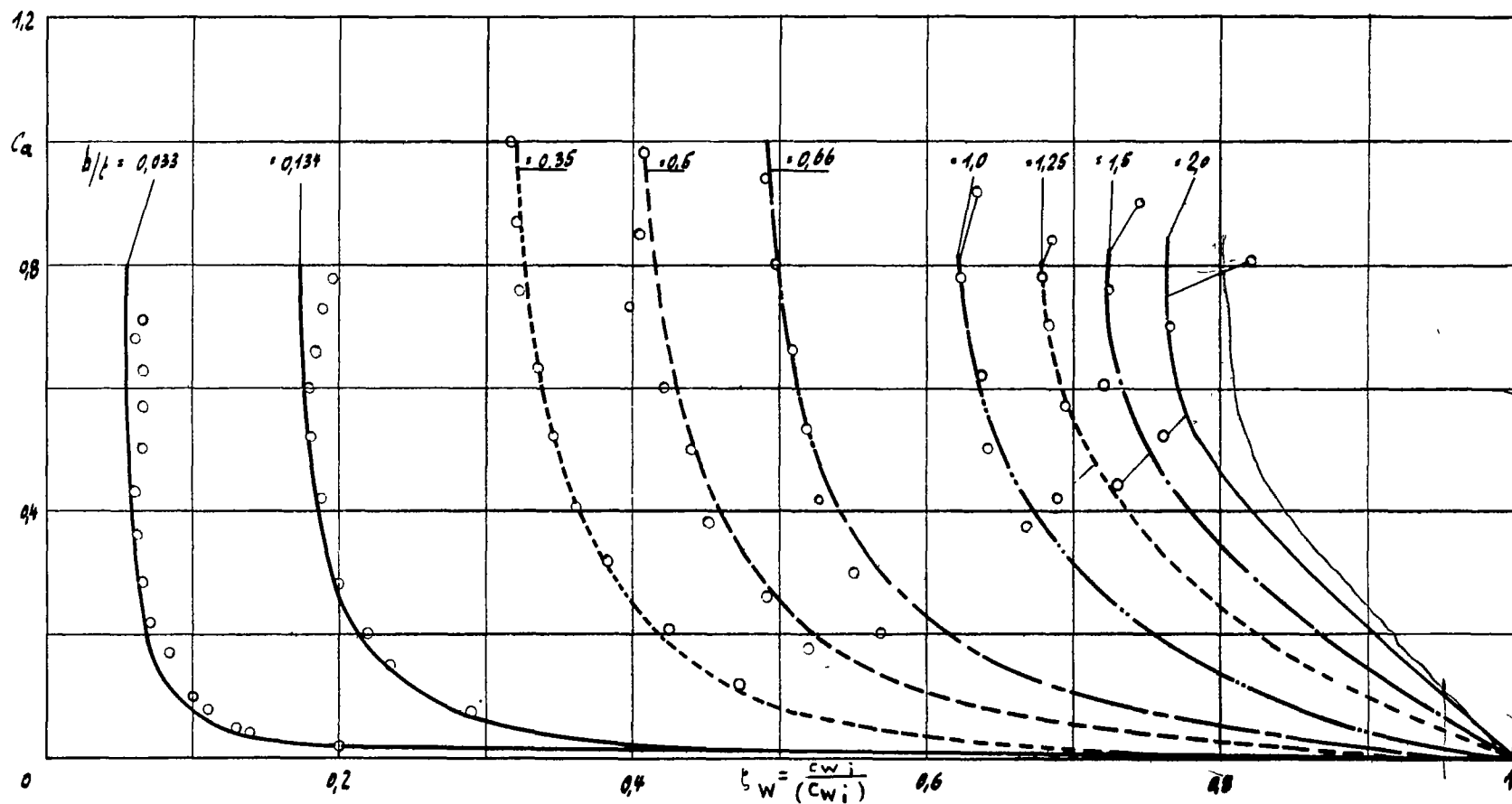


Figure 7a.- Ratio $\xi_w = \frac{c_{w_i}}{(c_{w_i})}$ (ratio of the induced drag coefficient c_{w_i} , according to the theory modified for small span, to the induced drag coefficient (c_{w_i}) , according to the theory for large span wings) as a function of the lift coefficient C_a for various aspect ratios, based upon the measured relation between lift coefficient and angle of attack.

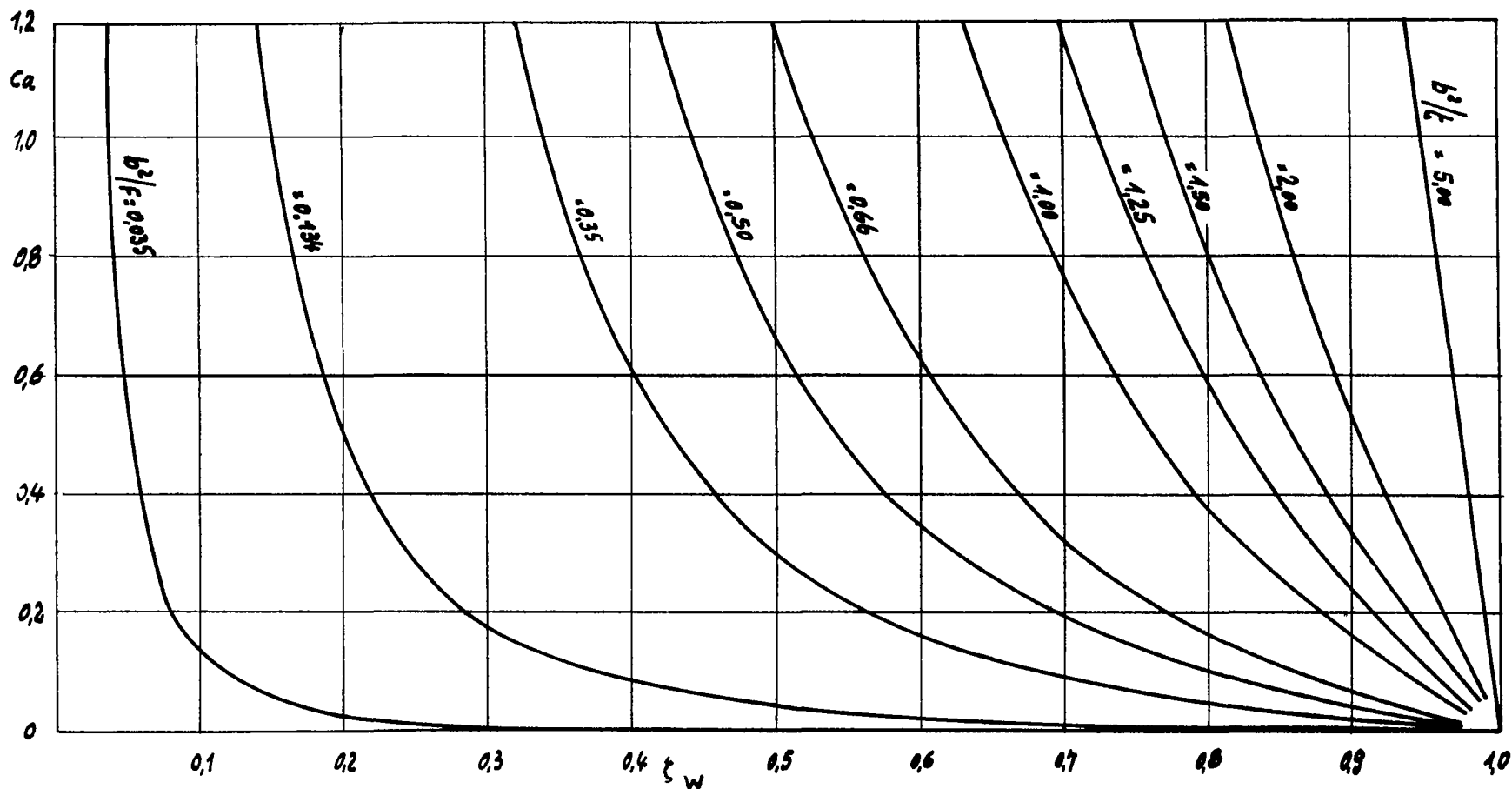


Figure 7b.- The agreement between the ratio $\xi_w = \frac{c_{w_i}}{(c_{w_i})}$ (ratio of the induced drag coefficient c_{w_i} , according to the theory modified for small span, to the induced drag coefficient (c_{w_i}) , according to the theory for large span wings), shown as a function of the lift coefficient c_a for various aspect ratios, and the calculated relation between lift coefficient and angle of attack.

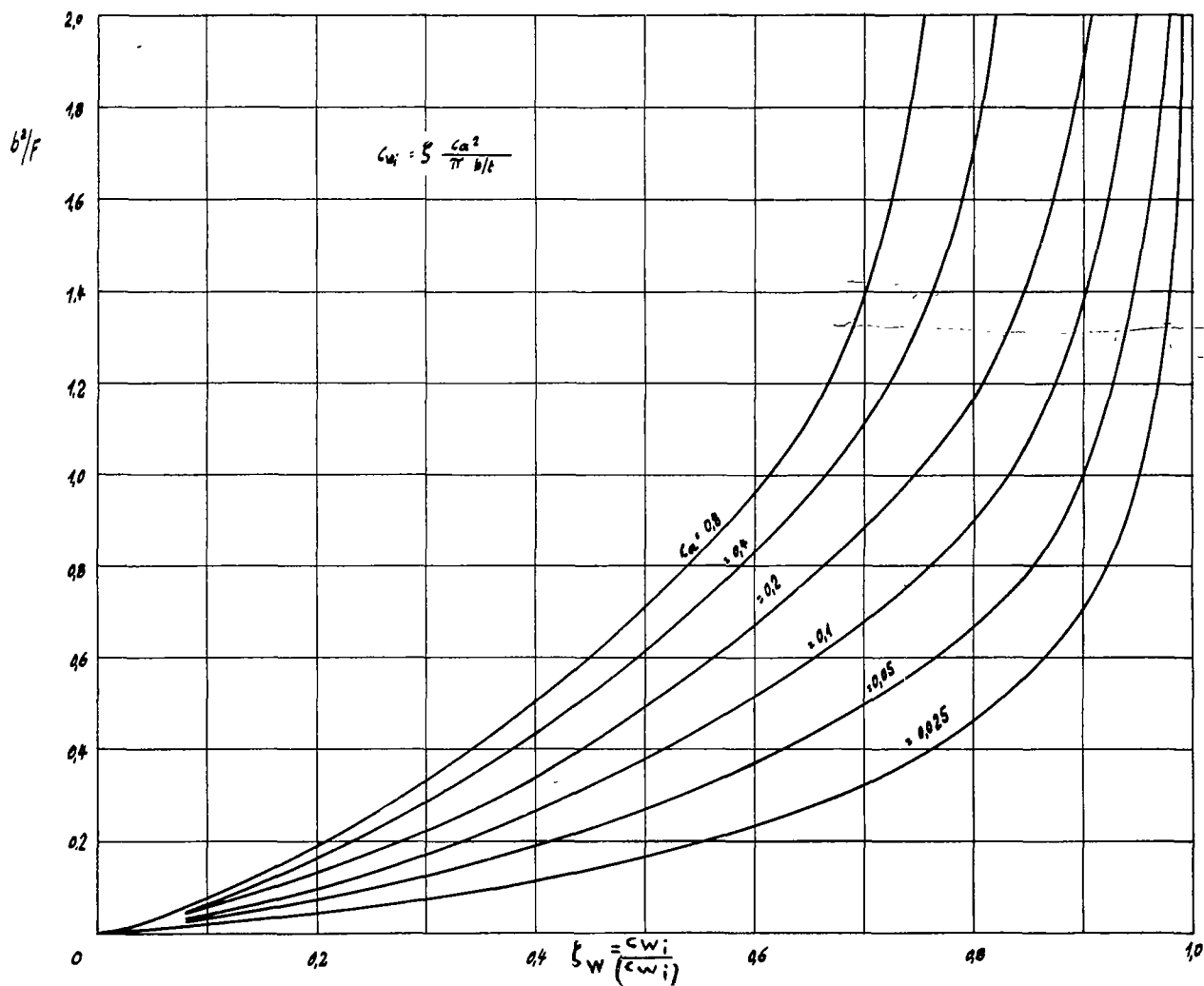


Figure 8a.- Ratio $\xi_w = \frac{c_{wi}}{c_{wi}}$ (ratio of the induced drag coefficient c_{wi} , according to the theory modified for small span, to the induced drag coefficient (c_{wi}) , according to the theory for large span wings), as a function of the aspect ratio for various lift coefficients based upon the measured relation between lift coefficient and angle of attack.

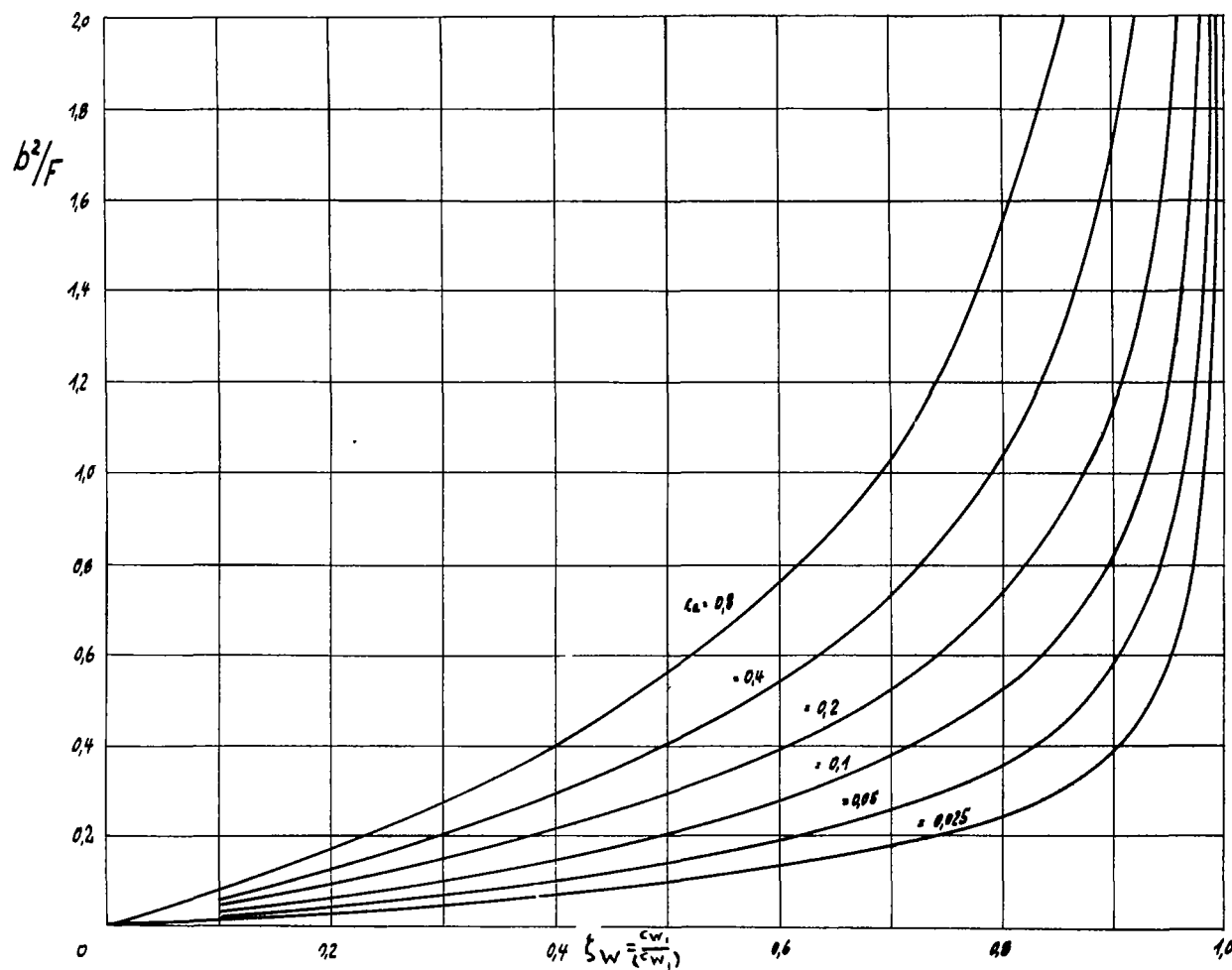


Figure 8b.- Ratio $\zeta_W = \frac{c_{wi}}{(c_{wi})_\infty}$ (ratio of the induced drag coefficient c_{wi} , according to the theory modified for small span, to the induced drag coefficient $(c_{wi})_\infty$, according to the theory for large span wings), as a function of the aspect ratio for various lift coefficients in agreement with the calculated relation between lift coefficient and angle of attack.

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